MESH IMPROVEMENT BY SUBDIVISION OF TETRAS INTO HEXES

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A tetra 4 mesh was transformed by subdividing each mesh into 4 hex 8 elements. The resulting hex mesh from such a transformation was found to give superior accuracy when compared to equivalent subdivisions of the tetra mesh. In the comparison of a rectangular beam model, it was found that the tetra mesh required at least 4 h=2 subdivisions to give the same accuracy as that of the transformed hex mesh. Similar improvements were found for the mesh for the well known socket problem.

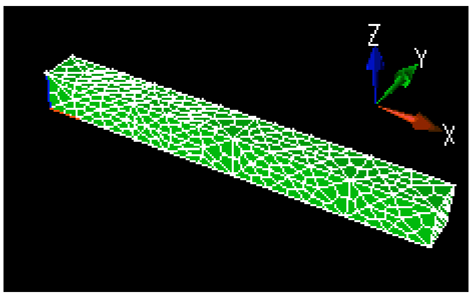

Fig. 1 Hex Mesh by Subdivision

mesh	Num. of nodes	Max Eq. Stress	Max. disp.	Max.Error norm
Tetra orig	98	240	0.00056	1.0
Tetra h=2	511	488	0.00091	0.91
Tetra h=4	3181	573	0.00114	0.63
Tetra h=8	22169	624	0.00125	NA
Hexa h~2	1274	688	0.00131	0.11

The results are surprising since the bad aspect ratios of the tetra corners have always been blamed for the poor performance of these elements. The same bad aspect ratios are now found in the hex elements. In the beam problem there are no complications from geometric curvature at the surfaces This permits us to examine the effects of error in the two elements as only a function of their truncation error. The effects of the truncation can be seen more clearly by considering the two dimensional case. Many writers have termed the truncation in the simplex triangular elements as second order because the terms in the first order is complete. But this is true not only along the edges of the triangle.

We assume $u = a_0 + a_1 x + a_2 y$

Along any other line emanating from the corner, we can substitute y=mx and obtain a truncation in the quadratic term in x. In the case of the rectangular element, the truncation is quadratic along the edges

We assume $u = a_0 + a_1 x + a_2 y + a_3 x y$

Along any other line emanating from the corner, we again assume that y=mx and obtain a truncation in the cubic term of x. We observe that the truncation order is enhanced as we move away from the edges. This explains the superiority of the quadrilateral elements (when we use isoparametric elements).

Finally, we consider the application of the error norm for these meshes in adaptive analysis. The error norm is a strain energy based local error. In order to give it a globally relative measure we weigh the error norm by its equivalent stress. This allows us to select the node with the maximum weighted error and subdivide the elements connected to that node. This improves the performance of the mesh substantially and reduces the number of nodes required for convergence in any downstream adaptive analysis. The maximum weighted error changed from 75 to 50 and the peak location moved to the end of the beam. The maximum stress changed from 240 to 488, the peak given by a full mesh subdivision. The number of nodes used for this refinement was 110 versus the 511 for the full subdivision.